problems. The other was the typical combination of a MUSIC angle estimator [6] and PDA association filter [1], which is computationally expensive. Monte Carlo simulations of 100 runs were carried out for each algorithm with various SNRs. The tracking success rates and mean square errors are summarised in Table 1. Park's algorithm showed a very poor tracking performance in the case of three crossing targets. It should be noted that the proposed algorithm, which retains the simple structure of Park's algorithm, gives comparable results to those of the computationally expensive MUSIC (PDA) algorithm.

Conclusion: The proposed algorithm retains a simple structure and can avoid data-association problems. In addition, it produces a comparable tracking performance to that of the computationally expensive MUSIC algorithm with a PDA association filter.

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Simple expressions for worst and best case Cramér-Rao bounds for amplitude and phase estimation of closely spaced cisoids

E. Dilaveroğlu

Simple expressions are presented for the worst and best case Cramér-Rao bounds for the amplitude and phase estimation of two cisoids in the presence of complex white Gaussian noise. The expressions are valid in the sub-Rayleigh region where the difference between the critical values of the bounds becomes important.

Introduction: The Cramér-Rao bound (CRB) expressions for the amplitudes, phases, and frequencies of superimposed cisoids in the presence of complex white Gaussian noise are well documented (e.g. [1, 2]). For two cisoids, it is known that the CRBs strongly depend on the phase difference between the cisoids in the sub-Rayleigh interval (where the frequency separation between the cisoids is less than the resolution limit of the periodogram). In this region, it is thus important to determine the largest and the smallest values of the bounds as the phase difference is varied. The problem has been recently treated in [3] for frequency CRBs. An analytical solution to the problem requires closed-form (nonmatrix) expressions for the bounds which give the dependence of the bounds on the phase difference explicitly. In this Letter, we extend the solution to the other CRBs. Simple expressions for the largest and the smallest amplitude and phase bounds are given. The expressions are valid in the small-frequency-separation region.

Model description: We consider the data model

$$y(t) = \sum_{i=1}^{2} \alpha_i \exp[j(\omega_i t + \varphi_i)] + e(t) \qquad t = 1, ..., N \quad (1)$$

where $j = \sqrt{-1}$, α_i is the amplitude, φ_i is the phase, ω_i is the frequency of the ith cisoid, $i = 1, 2$, $e(t)$ is zero-mean complex white Gaussian noise with variance σ^2, and N is the total number of data samples.

Critical bound expressions: It is tedious but straightforward to obtain nonmatrix expressions for the amplitude and phase CRBs for the model in eqn. 1 (e.g. from the results in [3]). Moreover, the values of the phase difference at which the bounds attain their maximum and minimum values can easily be determined from the expressions.

Having obtained the expressions for the largest and smallest amplitude and phase bounds, we express the frequency-separation-dependent functions that appear in the expressions in terms of Taylor series about $\delta\omega = 0$ where $\delta\omega$ denotes the frequency separation. After algebraic manipulations, we find for small $\delta\omega$ that

$$[CRB]_{worst} = 201600\left[\frac{N^6}{(N^2-1)(N^2-4)(N^2-9)}\right]\frac{\sigma^2}{N}(N\cdot\delta\omega)^{-6} + O\{(N\cdot\delta\omega)^{-4}\} \quad (2)$$

$$[CRB]_{best} = 360\left[\frac{N^4}{(N^2-1)(N^2-4)}\right]\frac{\sigma^2}{N}(N\cdot\delta\omega)^{-4} + O\{(N\cdot\delta\omega)^{-2}\} \quad (3)$$

for estimating the amplitude α_i, and

$$[CRB]_{worst} = 12960\left[\frac{N^4(N+\frac{1}{2})(N+\frac{13}{9})}{(N^2-1)(N^2-4)(N^2-9)}\right]\frac{1}{SNR_i\cdot N}(N\cdot\delta\omega)^{-4} + O\{(N\cdot\delta\omega)^{-2}\} \quad (4)$$

$$[CRB]_{best} = 96\left[\frac{N^2(N+\frac{1}{2})(N+\frac{11}{8})}{(N^2-1)(N^2-4)}\right]\frac{1}{SNR_i\cdot N}(N\cdot\delta\omega)^{-2} + O\{1\} \quad (5)$$

for estimating the phase φ_i. Here, SNR_i denotes the signal-to-noise ratio for the ith cisoid, $SNR_i = \alpha_i^2/\sigma^2$.

Note that the inverse power dependence of the asymptotic bounds on $\delta\omega$ is through $(N\cdot\delta\omega)$, indicating that the bounds will be large, provided that $(N\cdot\delta\omega)$, rather than $\delta\omega$, is small. This shows the importance of having a sufficient number of data samples when $\delta\omega$ is small.

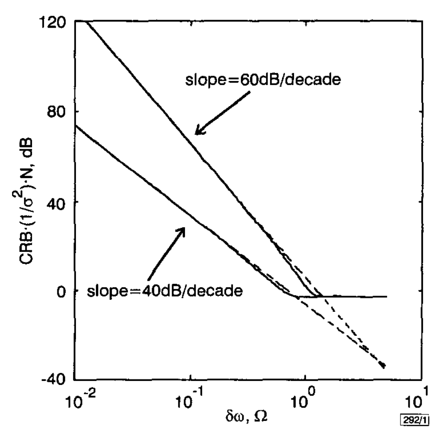

Fig. 1 *Largest and smallest normalised exact and asymptotic CRB for amplitudes of two cisoids having frequency separation* $\delta\omega$, *observed by* $N = 10$ *uniformly spaced samples*

——— exact
– – – – asymptotic

Eqns. 2 and 3 show that the largest amplitude CRB is proportional to $(N \cdot \delta\omega)^{-6}/N$ while the smallest amplitude CRB is proportional to $(N \cdot \delta\omega)^{-4}/N$ for small frequency separations $\delta\omega$. Eqns. 4 and 5 show that the largest phase CRB is proportional to $(N \cdot \delta\omega)^{-4}/N$ while the smallest phase CRB is proportional to $(N \cdot \delta\omega)^{-2}/N$ for small frequency separations $\delta\omega$.

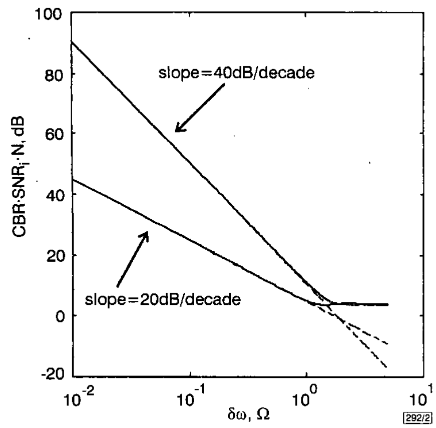

Fig. 2 *Largest and smallest normalised exact and asymptotic CRB for phases of two cisoids having frequency separation $\delta\omega$, observed by N = 10 uniformly spaced samples*

——— exact
– – – – asymptotic

Example: Consider the case of N = 10 data samples. Fig. 1 shows the largest and smallest amplitude bounds and Fig. 2 shows the largest and smallest phase bounds. The solid curves in the Figures depict the exact bounds while the dashed lines depict the asymptotic bounds. The horizontal co-ordinate in the Figures depicts the value of $\delta\omega/\Omega$, where Ω denotes the Rayleigh limit, $\Omega = 2\pi/N$. We see that the actual bounds closely follow the asymptotic bounds for small values of $\delta\omega/\Omega$. Note that the difference between the two limits of the bounds is large in the interval $\delta\omega/\Omega < 1$ indicating the strong dependence of the bounds on the phase difference in this region. For $\delta\omega/\Omega > 1$, the difference becomes small and the dependence of the bounds on the phase difference may be neglected.

Conclusions: Simple approximate expressions have been presented for the largest and the smallest amplitude and phase CRBs for two cisoids in the presence of additive complex white Gaussian noise. The expressions are valid in the sub-Rayleigh region, which is the region of most interest. These expressions also provide a better understanding of the behaviour of the bounds in this region.

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Suggested new bit rates for ITU-T G.723.1

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A 4.5/3.9kbit/s coder modified from the ITU standard G.723.1 is presented. Owing to the ease of modification and comparable speech quality, it is suggested that the new rates be incorporated in the standard as new options for rate selection.

Introduction: Low bit rate speech coders have many applications in modern communication networks as well as in the consumer electronics industry. The ITU-T standard G.723.1 [1] is one of the most well-known examples, being widely used due to its dual rate characteristic and good speech quality at a rate as low as 5.3kbit/s. Methods for achieving lower bit rates are still being studied. The most difficult factor to overcome is the inevitable compromise between bit rate and speech quality. To reduce the bit rate while retaining good speech quality we attempted a simple modification of the ITU-T G.723.1 algorithm. The concept is to alternate the method of constructing the excitation between the two known methods, namely CELP (code excitation linear prediction) [2] and SELP (self-code excitation linear prediction) [3]. Although the purpose is to reduce the bit rate, a reduction in computational complexity arises as a by-product. Simple as it may seem, the modification leads to a significant increase in performance, as will be shown, in terms of bit rate reduction, while retaining good speech quality.

Basics: The encoder of the ITU-T G.723.1 coder generates information such as LPC coefficients, pitch lags, gains, and stochastic code/gain indices for the decoder to decode into synthesised speech. The information is computed through a short-term predictor, a long-term predictor, and a stochastic code book search algorithm. The theory behind it, and shared by most CELP based coders [3], is as follows: a human vocal cavity may be modelled by a linear all-pole system which, when excited by a periodic excitation signal, can generate voiced sounds. If this theory is, by itself, complete and the computation of LPC coefficients and pitch/gain is accurate then there would be no need for stochastic coding. It is because of the overly-simplistic nature of the model that stochastic coding is required. The stochastic element added in the excitation signal primarily compensates for the imperfections of the LPC model and the limitations of the pitch estimate. It does not to any great extent suggest that the stochastic behaviour of speech in the 0–4kHz frequency band is of importance. Our argument is that, if we can very accurately compute the LPC coefficients and pitch lags/gains, we will rely less on stochastic corrections and therefore less information will be needed. Based on this argument, we developed the following method.

First, we noted that a perfect LPC/pitch estimator is difficult to realise. However, the excitation which is used to excite the imperfect system can be designed to meet a certain standard. The fact that, in G.723.1, the excitation of a subframe is extended from the previous subframe and corrected later by the 'best match' between the target and the synthesised speech leads us to adopt the concept of the SELP method. In other words, if the excitation of the previous subframe is guaranteed to generate good speech quality of that subframe, then the extension of it with new pitch lag/gain updates of the current subframe should lead to the generation of speech quality comparable to that of the previous subframe. Certainly, this argument is based on the assumption of slow variation of the LPC coefficients between two adjacent subframes, which is legitimate considering that the LPC coefficients are interpolated within a frame in the standard G.723.1 algorithm.

Implementation: The actual implementation starts with one subframe of the standard coding in that the stochastic code search is performed and the stochastic code/gain indices are sent to the decoder. The excitation thus decoded is considered sufficient for generating good speech quality, which has been well accepted by G.723. 1 users. This is because the subframe coming immediately after the stochastic code/gain search is bypassed and no information is sent. The excitation signal of this subframe is constructed solely through SELP, i.e. deriving from the previous subframe without resorting to the stochastic codebook. Because of the 'best matching policy' of the pitch lag/gain search the synthesised