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Simple general expression for Cramér–Rao bound in presence of nuisance parameters

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A simple expression for computing the Cramér–Rao bound for a subset of unknown parameters in a data model regarding the remaining set of parameters as nuisance is presented. The expression is valid whenever the Fisher information matrix for the whole set is nonsingular.

Introduction: It is well known that the Cramér–Rao bound (CRB) matrix provides a lower bound on the covariance matrix of unbiased estimators of unknown parameters in a data model. For the case in which some of the parameters are of main interest (the principal parameters) while the other parameters may be treated as nuisances (the nuisance parameters), it becomes important to obtain a compact formula for the CRB matrix corresponding only to the principal parameters.

In recent papers [1, 2], an interesting method has been suggested to compute the CRB matrix for the principal parameters in a standard data model which is frequently encountered in the signal processing area. The method is based on a reparametrisation of the original model in such a way that the CRB matrix in the new parametrisation becomes block diagonal, thus decoupling the principal parameters from the nuisance parameters [2].

In this Letter, we generalise the method and show that it can be applied to any data model having a nonsingular Fisher information matrix (FIM). As an example, we consider the data models in [1, 2] and show that our general formula reduces to their known expression.

CRB for principal parameters: Let $\theta \in R^{d\times 1}$ be the vector of unknown parameters in a data model such that

$$\theta = [\,\theta_1^T \quad \theta_2^T\,]^T \tag{1}$$

where $\theta_1 \in R^{d_1\times 1}$ is the vector of nuisance parameters and $\theta_2 \in R^{(d-d_1)\times 1}$ is the vector of principal parameters. Let J be the FIM corresponding to θ. We assume that J is nonsingular (positive definite). J can then be written as

$$J = \Phi^T\Phi \tag{2}$$

where the matrix $\Phi \in R^{p\times d}$ has full column rank. Partition Φ as

$$\Phi = [\,\Phi_1 \quad \Phi_2\,] \tag{3}$$

where $\Phi_1 \in R^{p\times d_1}$ and $\Phi_2 \in R^{p\times(d-d_1)}$.

The CRB matrix for the parameter vector θ equals J^{-1}. Now assume that we reparameterise the data model via the transformation

$$\theta_{new} = F\theta \tag{4}$$

where

$$F = \begin{bmatrix} I & (\Phi_1^T\Phi_1)^{-1}\Phi_1^T\Phi_2 \\ 0 & I \end{bmatrix} \tag{5}$$

Note that there is a one-to-one mapping between the original parameter vector θ and the new one in (4). Also note that in the new parametrisation, the principal parameters are left unchanged while the nuisance parameters (which are of no interest to us) are transformed.

The CRB for the new parameter vector in (4) is related to the original CRB as follows:

$$\begin{aligned} \mathrm{CRB}_{new} &= F\,\mathrm{CRB}F^T \\ &= FJ^{-1}F^T = \left\{(F^{-1})^T J(F^{-1})\right\}^{-1} \\ &= \left\{(\Phi F^{-1})^T(\Phi F^{-1})\right\}^{-1} \end{aligned} \tag{6}$$

It is not difficult to see that

$$F^{-1} = \begin{bmatrix} I & -(\Phi_1^T\Phi_1)^{-1}\Phi_1^T\Phi_2 \\ 0 & I \end{bmatrix} \tag{7}$$

and that

$$\begin{aligned} \Phi F^{-1} &= [\,\Phi_1 \quad \Phi_2\,]F^{-1} \\ &= [\,\Phi_1 \quad \Phi_2 - \Phi_1(\Phi_1^T\Phi_1)^{-1}\Phi_1^T\Phi_2\,] \\ &= [\,\Phi_1 \quad (I - P_{\Phi_1})\Phi_2\,] \end{aligned} \tag{8}$$

where $P_{\Phi_1} = \Phi_1(\Phi_1^T\Phi_1)^{-1}\Phi_1^T$ is the projection matrix onto the column space of Φ_1 and $(I - P_{\Phi_1})$ is the projection matrix onto the orthogonal complement of that space. Inserting (8) in (6) gives

$$\mathrm{CRB}_{new} = \begin{bmatrix} \Phi_1^T\Phi_1 & 0 \\ 0 & \Phi_2^T(I - P_{\Phi_1})\Phi_2 \end{bmatrix}^{-1} \tag{9}$$

Note that the new CRB matrix is block diagonal and the principal parameters are decoupled from the other parameters. Thus, the CRB matrix for the principal parameter vector θ_2, which we denote by CRB_{θ_2}, is

$$\mathrm{CRB}_{\theta_2} = \left\{\Phi_2^T(I - P_{\Phi_1})\Phi_2\right\}^{-1} \tag{10}$$

This is the sought-after compact expression for the CRB matrix corresponding to the principal parameters. The expression is valid whenever the original FIM is positive definite. As an illustration, we use the result in (10) to compute the CRB for the principal parameters in the data model considered in [1, 2] in the following.

Example: Consider the following data model:

$$y = A(\omega)x + e \tag{11}$$

where $y \in C^{m\times 1}$ is the vector of observations, $e \in C^{m\times 1}$ is a noise vector, $x \in C^{n\times 1}$ is the vector of nuisance parameters, $\omega \in R^{n\times 1}$ is the vector of principal parameters, and the matrix $A(\omega) \in C^{m\times n}$ has the following special structure:

$$A(\omega) = [a(\omega_1), \ldots, a(\omega_n)] \tag{12}$$

where $\omega = [\omega_1, \ldots, \omega_n]^T$ and the vector $a(s)$ is a generic vector specified by a real parameter s.

The matrix $A(\omega)$ is assumed to have full column rank and the noise vector e to be circularly Gaussian distributed with mean zero and covariance matrix $\sigma^2 I$.

Let $(\cdot)_r$ and $(\cdot)_i$ denote the real and imaginary part, respectively. The FIM for the signal parameter vector $[x_r^T\; x_i^T\; \omega^T]^T$ in (11) is given by (see [2])

$$J = \frac{2}{\sigma^2}(G^H G)_r \tag{13}$$

where

$$G = [A \quad iA \quad D] \tag{14}$$

$$D = [d(\omega_1)x_1, \ldots, d(\omega_n)x_n] \tag{15}$$

$$d(\omega_j) = \left.\frac{da(s)}{ds}\right|_{s=\omega_j} \tag{16}$$

and where $x = [x_1, \ldots, x_n]^T$. (We have omitted the argument of $A(\omega)$ for notational convenience.)

Now let

$$\Phi = \frac{\sqrt{2}}{\sigma}\begin{bmatrix} G_r \\ G_i \end{bmatrix} = \frac{\sqrt{2}}{\sigma}\begin{bmatrix} A_r & -A_i & D_r \\ A_i & A_r & D_i \end{bmatrix} \tag{17}$$

Then

$$\Phi_1 = \frac{\sqrt{2}}{\sigma}\begin{bmatrix} A_r & -A_i \\ A_i & A_r \end{bmatrix}, \quad \Phi_2 = \frac{\sqrt{2}}{\sigma}\begin{bmatrix} D_r \\ D_i \end{bmatrix} \tag{18}$$

and

$$\begin{aligned} \mathrm{CRB}_\omega &= \{\Phi_2^T(I - P_{\Phi_1})\Phi_2\}^{-1} \\ &= \{\Phi_2^T\Phi_2 - \Phi_2^T\Phi_1(\Phi_1^T\Phi_1)^{-1}\Phi_1^T\Phi_2\}^{-1} \\ &= \frac{\sigma^2}{2}\Bigg\{(D^H D)_r - [(D^H A)_r - (D^H A)_i] \\ &\quad \times \begin{bmatrix} (A^H A)_r & -(A^H A)_i \\ (A^H A)_i & (A^H A)_r \end{bmatrix}^{-1} \begin{bmatrix} (A^H D)_r \\ (A^H D)_i \end{bmatrix}\Bigg\}^{-1} \end{aligned} \tag{19}$$

which is the result in [2, eqn. 8].

Conclusion: We have derived a compact expression for computing the CRB for the principal parameters in a data model, treating the other parameters in the model as nuisance parameters. The expression is applicable whenever the FIM for the data model is nonsingular.

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Simple nonlinear controller to reduce line and load disturbances in HBCC converter

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A simple nonlinear control scheme that reduces line and load disturbances in the half-bridge complementary-control (HBCC) converter is proposed. The control scheme is devised by imposing a desired linear dynamic behaviour on the output voltage, by means of input-output feedback linearisation techniques.

Introduction: Nowadays, new integrated circuits (ICs) require power supplies that can deliver low voltages with tight regulation and fast transient response [1, 2]. The use of the half-bridge complementary-control (HBCC) converter with synchronous rectifiers is a suitable solution to supply low voltage and high current to new ICs. The main advantages of this topology can be summarised as follows [3]. (i) The input voltage of the converter is high; therefore, conduction losses can be reduced, since the input current is low. (ii) The waveforms for self-driven synchronous rectifiers are optimum, and the performance of the synchronous rectification stage is very simple. (iii) The output filter is small, since the input voltage at this stage is always positive and its waveform is between two positive levels that are close to the output voltage.

In a previous work, a control scheme based on a small-signal model of this converter was proposed [4]. However, the main disadvantage of this control is its high sensitivity to input voltage disturbances, thus making it necessary to use a pre-regulator.

In this Letter, we present a nonlinear control for this structure capable of overcoming this drawback. The controller allows one to decouple the output filter dynamics from the input voltage and to impose a desired linear dynamic behaviour on the output voltage.

Circuit description and nonlinear model: Fig. 1 shows the HBCC power stage. It consists of a half-bridge inverter loaded by an isolated synchronous rectifier and a lowpass filter. The converter can be described by the following bilinear model:

$$C_{eq}\frac{dv_{C2}}{dt} = C_1\frac{dE}{dt} + i_m + ni_L(2u - 1) \tag{1}$$

$$L_m\frac{di_m}{dt} = Eu - v_{C2} \tag{2}$$

$$L\frac{di_L}{dt} = nEu - nv_{C2}(2u - 1) - v_o \tag{3}$$

$$C\frac{dv_o}{dt} = i_L - \frac{v_o}{R} \tag{4}$$

where C_{eq} is $C_1 + C_2$, and u is the control variable, which takes the following values: $u = 1$ (when S1 is ON and S2 is OFF), and $u = 0$ (when S1 is OFF and S2 is ON).

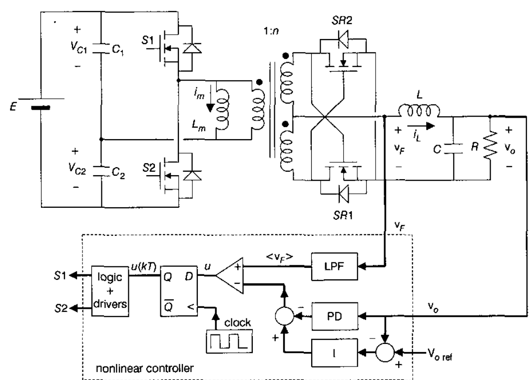

Fig. 1 *Power stage of HBCC converter and block diagram scheme of proposed nonlinear controller*

Control design: Our aim in this Section is to introduce input-output feedback linearisation [5, 6] in order to deduce a nonlinear controller that decouples the output voltage dynamics from the input voltage. To deduce the controller description which stabilises the external dynamics, it is necessary to find a differential equation of the output voltage v_o in which the control variable u explicitly appears. By using (3) and (4) and taking the average value over one switching period, the output voltage dynamics can be found:

$$LC\frac{d^2\langle v_o\rangle}{dt^2} + \frac{L}{R}\frac{d\langle v_o\rangle}{dt} + \langle v_o\rangle = \langle nEu - nv_{C2}(2u - 1)\rangle \tag{5}$$

where the symbol $\langle\ \rangle$ denotes the average value.

To linearise the output voltage dynamics, we propose to implement the controller by means of the following equality:

$$-k_p\langle v_o\rangle + k_i\int_{-\infty}^{t}(V_{oref} - \langle v_o\rangle)d\tau - k_d\frac{d\langle v_o\rangle}{dt} = \langle nEu - nv_{C2}(2u - 1)\rangle \tag{6}$$

From (5) and (6), the closed-loop output voltage dynamics then yields:

$$LC\frac{d^3\langle v_o\rangle}{dt^3} + \left(\frac{L}{R} + k_d\right)\frac{d^2\langle v_o\rangle}{dt^2} + (1 + k_p)\frac{d\langle v_o\rangle}{dt} + k_i\langle v_o\rangle = k_iV_{oref} \tag{7}$$

As it can be seen, the above equation, which is the external dynamics, is decoupled from the internal dynamics (i_m and v_{C2}). The stability of the internal dynamics can be easily proved by means of the zero dynamics approach [5]. Moreover, the zero dynamics is independent from the control parameters, and, consequently, no design constraints are derived from this analysis. Thus, by choosing k_p, k_i, and k_d, we can obtain a proper dynamics of the output voltage.

Controller implementation: Fig. 1 also shows the block diagram of the proposed nonlinear control. This controller implements (6), which